EE 313 Linear Systems and Signals

Convolution of Two Causal Exponential Sequences

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Case #1. $y[n] = x_1[n] * x_2[n]$ where $x_1[n] = a^n\, u[n]$ and $x_2[n] = b^n\, u[n]$ and $a \neq b$.

$$y[n] = (a^n u[n]) * (b^n u[n])$$

$$y[n] = \sum_{m=-\infty}^{\infty} \left(a^m u[m]\right)\left(b^{n-m} u[n-m]\right)$$

For $u[m]$ to be 1, $m \geq 0$. For $u[n\text{-}m]$ to be 1, $n - m \geq 0$ or equivalently $m \leq n$.
For $n \geq 0$, the limits of summation are $0 \leq m \leq n$.

$$y[n] = \sum_{m=0}^{n} a^m b^{n-m} \quad \text{for } n \geq 0$$

The term of n inside the summation does not depend on m and can be pulled out.

$$y[n] = b^n \sum_{m=0}^{n} \left(\frac{a}{b}\right)^m \quad \text{for } n \geq 0$$

We can simplify the above summation using an identity on page 887 in Roberts' *Signals and Systems* book:

$$y[n] = b^n \frac{1 - \left(\frac{a}{b}\right)^{n+1}}{1 - \frac{a}{b}} \quad \text{for } n \geq 0$$

We can multiply numerator and denominator by b, and multiply the b^n term through numerator to obtain the following result:

$$y[n] = \frac{b^{n+1} - a^{n+1}}{b - a} u[n]$$

Case #2. $y[n] = x_1[n] * x_2[n]$ where $x_1[n] = b^n\, u[n]$ and $x_2[n] = b^n\, u[n]$.
The first four steps are the same as in the above. We can then substitute $a = b$:

$$y[n] = b^n \sum_{m=0}^{n} 1^m \quad \text{for } n \geq 0$$

With $1^m = 1$, we are summing 1 for $(n+1)$ times when $n \geq 0$, which gives us

$$y[n] = (n+1) b^n u[n]$$